Stock Grants as a Commitment Device *

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Abstract

A large and increasing fraction of the value of executives' compensation is accounted for by security grants. It is often argued that the optimal compensation contracts characterized in the theoretical literature can be implemented by means of stock or option grants. However, in most cases the optimal allocation can be implemented simply by a contingent sequence of cash payments. Security awards are redundant. In this paper we develop a dynamic model of managerial compensation where neither the firm nor the manager can commit to long-term contracts. We show that, in this environment, if stock grants are not used, then the optimal contract collapses to a series of short term contracts. When stock grants are used, however, nonlinear intertemporal schemes can be implemented to achieve better risk-sharing and greater firm value.

Key words. Moral Hazard, Optimal Contracts, CEO Compensation, Stock Grants.

JEL Codes: D21, D82, G32.

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1 Introduction

The theoretical literature on optimal managerial compensation with moral hazard has long established that current and deferred compensation should be made contingent on the value of the firm. Managers should be paid more when shareholder value is higher, both in the current period and in the future. The compensation schemes that we observe in use typically consist of current cash compensation, stock and option grants, and promises of future cash compensation. It is often argued that stock and option grants are natural means to implement contingent deferred compensation. However, in the case of most models the optimal allocation can be implemented simply by a sequence of contingent cash payments.[1] Security awards are redundant instruments, in the sense that they do not offer any advantage over a contingent sequence of cash outlays.

In this paper we show that an exclusive role for securities grants arises in environments where the enforcement of contracts is limited, so that firms cannot commit to follow up on promises of cash compensation. In this case, firms can motivate their employees with promises of deferred cash compensation only to the extent that such promises are self-enforcing. Securities grants can provide a partial solution to this inefficiency, as firms find it harder to renege on payments to shareholders than on cash payments to employees. If we abstract from enforcement, the schedule of contingent cash-flows provided by a given security grant can be awarded to an executive by means of a contract that explicitly specifies the payment corresponding to each state of nature. However, we argue below, companies' (shareholders') ability to renege on the latter form of compensation is much greater. An executive will attach a higher value to the security grant, implying that compensating management with stock and/or options will increase shareholder value.

A *vested*[2] stock grant is a sure claim to a risky cash flow, as it can be easily exchanged for cash once the eventual selling restrictions have expired. The same can be said of a vested option grant, as it can be exchanged for a non-negative cash flow at the exercise date. This is not the case for deferred cash payments, even when they are part of an explicit contract. While systematic studies have not been conducted yet,

[1]This is the case for both static models such as Haubrich (1994), Holmstrom (1979), and Garen (1994), and dynamic models such as Wang (1997) and Clementi and Cooley (2000).

[2]A stock or option grant vests when the grantee acquires ownership of the securities. Further restrictions, however, may hinder him from selling the stock or exercising the options. In the remainder of this paper we will say that a grant is unrestricted when the shares (options) acquired by the grantee can be freely disposed of (exercised).

there is ample anecdotal evidence that firms do default on promises of cash payments to employees, let them be wages, or medical and insurance benefits, or pensions, or severance pay. Shleifer and Summers (1988) have argued that, in the case of many corporate acquisitions, a large fraction of the increase in the target's shareholder value is due the acquirer's ability to renege on employees' long-term compensation contracts. The ongoing US Airways reorganization process indicates that Chapter 11 of the Bankruptcy Code allows corporations to default on their long-term obligations towards current and past employees, in order to preserve their viability. Recently the judge in charge of the case has allowed US Airways to terminate the pilots' pension plan as a necessary step to avoid liquidation. The recent boom in executive compensation litigation provides further support for our hypothesis that the enforcement of compensation contracts is imperfect. Utz (2001a,b) gives an account of the most frequent causes of litigation and illustrates them with a short series of cases. Among the most common disputes are those that concern the degree to which an employer may amend or terminate a severance pay plan, therefore undermining the employee's ability to cash on the employer's promise.[3] A severance pay plan is a typical example of what we refer to as an explicit promise of deferred cash compensation. We interpret the large volume of severance pay litigation documented by Utz as a sign that enforcement of such promises is imperfect and that employers successfully attempt to renege on them.[4]

We build a simple two-period model of hidden action in which neither the firm (principal) nor the manager (agent) can commit to long-term compensation contracts. At the beginning of the second period the two agents will execute the continuation of the long-term contract only if it provides each of them with payoffs greater than their outside options. The remaining assumptions are standard. The probability distribution of the firm's profits depends in a natural way on the (unobservable) effort exerted by the manager. The principal is risk-neutral, while the agent is risk-averse

[3]According to Utz (2001b), vesting standards dictated by the Employee Retirement Income Security Act do not apply to the typical severance plan. For this reason, an employer's ability to terminate or amend the plan is largely unrestricted, except to the extent that the terms of the plan itself restrict that right.

[4]One could argue that establishing an escrow account would allow firms to commit to deliver on at least some of the promises of deferred cash compensation. After all, this is similar to what happens in some European countries, where firms face tight restrictions on the use of funds accumulated to cover pension benefits and severance pay. However, as long as external finance is more expensive than internally generated funds, immobilizing funds in an escrow account is inefficient. This is the reason why in the same European countries there is pressure towards relaxing the restrictions mentioned above.

and has disutility from effort.

If the compensation consists of cash payments only, then it is easy to show that the optimal long-term contract collapses to a sequence of two independent static contracts. This occurs because regardless of the profit realization, the firm will not deliver to the manager a payoff greater than his outside value. In fact any larger payoff would result in the firm breaching the contract and hiring a new executive.[5] On the other hand, the manager will choose to quit whenever the continuation of the long-term contract promises less than his outside value.

Things are different if at the end of the first period the firm can grant stock to the manager and the size of the grant can be made contingent on firm performance. Suppose that the owners breach the contract and fire the manager. The cost of replacing him is now higher. In fact it equals the cost of hiring the substitute plus the dilution of shareholder value induced by the increase in the number of shares outstanding. For this reason, the firm can credibly commit to deliver to the manager in period 2 a payoff higher than his outside value. In equilibrium the firm will take up this opportunity, as it allows for better risk-sharing, and thus for a decrease in the cost of delivering a given expected utility to the manager. Under our assumptions on the market structure, this lower cost translates one to one into higher shareholder value.

The evidence gathered by Utz (2001b) shows that in reality enforcement problems arise also when securities are used, if the vesting of a stock or option grant is conditional on the cause of the employee's termination of employment. The reason is that courts have a hard time verifying the actual reason for termination.[6] In our model shareholder value is larger when vesting can be made contingent on the type of separation (i.e. when the courts can verify the reason of the termination). In particular, shareholder value is maximal if vesting is denied when the manager quits. However, we show that the introduction of securities grants always increases shareholder value, even when enforcement problems do not allow for vesting to be contingent.

[5]Throughout the paper we assume that breaching the compensation contract is costless. This assumption is made for the sake of simplicity and can be dispensed with. Our results still hold when we relax it, as long as the cost of breaching is not too large.

[6]For example, Utz (2001b) argues that courts are often called to determine whether an employee's termination of employment was of a type causing the employee's options to vest. In the case of Tredway v. Merck & Co. the plaintiff refused to transfer to a 50/50 joint venture of Merk with another organization, and took instead a job with an unrelated employer. The employee had received stock options, which were not vested when he terminated his contract. The court determined that this particular type of separation was not among the ones contemplated in the stock option plan, and therefore ruled in favor of the employer, denying vesting of the option grant.

There is very little (if any) theoretical work investigating the conditions under which awarding securities is actually optimal. Since the current US tax code and FASB standards discriminate across different means of compensation, it is likely that tax and accounting considerations play an important role in shaping employees' compensation packages.[7] In this paper we abstract completely from such considerations, with the purpose of isolating the role of limited enforcement.

The research on the optimal design of securities grants is also in its infancy. Aseff and Santos (2002) characterize the optimal stock option grant in a otherwise standard hidden action model. Acharya et al. (2000) investigate the optimality of resetting strike prices on previously-awarded option grants. In contrast to our work, in both of these papers compensating the manager by means of securities is suboptimal, in the sense that the use of contingent cash compensation would increase shareholder value.

The remainder of the paper is organized as follows. The model is introduced in Section 2. In Section 3 we show how to solve for the optimal long-term compensation contract with stock. In Section 4 we characterize the optimal compensation policy and we show that the inclusion of stock grants in the compensation package increases shareholder value. Section 5 considers the case in which the firm awards the CEO call options instead of stock. Section 6 concludes.

2 The Model

There are two periods: $t = 1, 2$. We consider the problem of a firm that needs a manager to operate in each period. There are many equally skilled individuals, each of whom can be the firm's manager. The firm is risk neutral and maximizes expected discounted dividends. The manager's preferences are described by the utility function

$$H(c_t, e_t) = u(c_t) - e_t,$$

where c_t and e_t denote time t consumption and effort, respectively. We assume that $u : \Re^+ \to \Re$ and that $u(\cdot)$ is twice continuously differentiable, strictly increasing, and strictly concave. All agents discount future utility at the same rate δ, $\delta \in (0, 1)$.

[7] See Lipman (2001) for a readable but comprehensive account of the tax and accounting treatment of the different components of managerial compensation. As an example of differential tax treatment, the compensation originated by option grants that qualify as Incentive Stock Options according to IRS guidelines is taxed at the long-term capital gain tax rate, which is lower than the marginal income tax rate that applies to cash compensation. On the accounting side, it is well known that, differently from cash compensation, the award of stock options does not generate any charge in the income statement.

We assume that for all t, $e_t \in \{0, a\}$, with $a > 0$. Further, let π_t denote the firm's profit at time t (gross of manager's cash compensation). We assume that for all t, $\pi_t \in \{\pi_H, \pi_L\}$, where $0 < \pi_L < \pi_H$, and that $prob(\pi_t = \pi_H \mid e = a) = \overline{\rho}$, $prob(\pi_t = \pi_H \mid e = 0) = \underline{\rho}$, with $\overline{\rho} > \underline{\rho} > 0$.

The level of the manager's effort is not observable to the firm and thus constitutes private information for the manager. We assume that if a manager doesn't work in a given period, then he receives a constant consumption c^*. This implies that his period reservation utility is $\omega = u(c^*)$.

At the beginning of period 1, the firm makes a take-it-or-leave-it offer to the manager (i.e. to one of the many identical candidates for the job). The offer consists a long-term compensation contract. It is long-term, in the sense that period-2 compensation is allowed to depend on both periods' profit realizations.

Definition 1 *A long-term compensation contract consists of contingent period-1 cash payments* $\{w_{1i}\}_{i=H,L}$*, period-1 contingent stock grants* $\{s_i\}_{i=H,L}$*, and period-2 contingent cash payments* $\{w_{2ij}\}_{i,j=H,L}$*.*[8]

The scalar s_i ($s_i \in [0, 1]$) denotes the fraction of equity that is granted to the manager at the end of period 1, contingent on the realization of state i.

We assume that both parties have limited commitment to the contract, in the following sense. At the beginning of period 2, both the firm and the manager can unilaterally decide to breach the contract at no pecuniary cost. They will do so if and only if the continuation values implied by the contract are lower than the values of their outside opportunities. If the contract is breached, neither party will fulfill his period-2 contractual obligations. The existing manager will become unemployed and the firm will have to hire another.

Notice that we do require period-by-period commitment. In other words, the beginning of period 2 is the only time in which the contract can be breached.[9] We say that the manager is fired whenever it is the firm that breaches the contract. Alternatively, if the contract terminates because of the manager's decision, we say that he quit.

[8]When clarity of exposition is not at stake, time indices will be suppressed.

[9]Our assumption of limited commitment is similar to those used by Phelan (1995) and Kocherlakota (1996) among others in the dynamic contracting literature. Phelan (1995) studies a model of moral hazard where commitment is one-sided. Kocherlakota (1996) considers two-sided limited commitment in a model of hidden endowment.

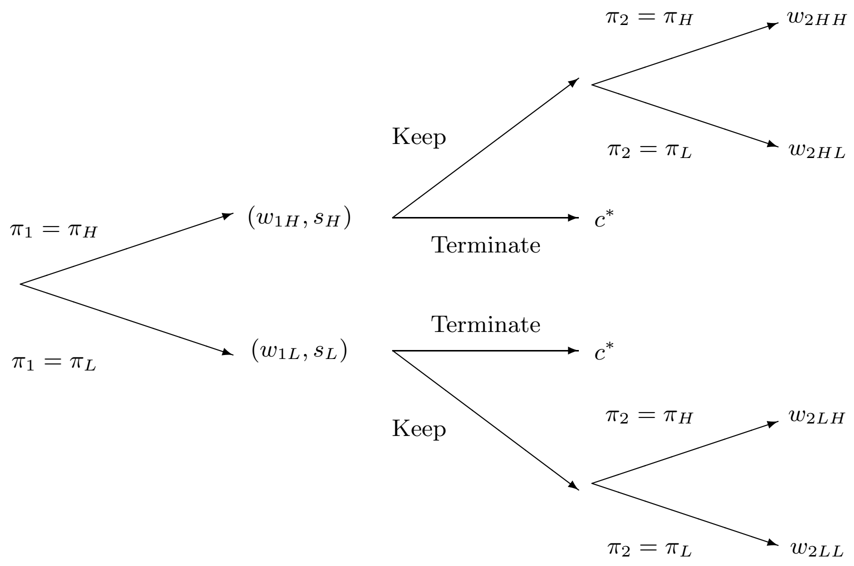

Figure 1: The Timing

2.1 Vesting and Sale Restrictions

We assume that in the case where the contract is not breached, vesting occurs at the beginning of period 2. However, the manager is restricted from selling the stock until period-2 uncertainty is resolved (i.e. until the end of the period). The same assumptions apply to the case in which the contract is terminated by the firm. Conditional on the manager quitting the job, we consider three possible scenarios, identified as A, B, and C, respectively. In scenarios A and B, the stock grant still vests at the beginning of period 2. In scenario A the manager is allowed to sell upon vesting (i.e. before period-2 uncertainty is resolved). In case B instead, he is restricted from selling until the end of the period (i.e. until period-2 uncertainty is resolved). Finally, in case C, the manager does not retain the stock if he decides to quit the firm. Vesting is denied.

3 Optimal Contracting

We solve for the optimal contract by backward induction. At the beginning of period 2, a stock holding s and a period-2 contingent cash payment (w_H, w_L) imply a level of expected utility U for the manager. For every utility level U, we determine the pair $(w_H(U,s), w_L(U,s))$ that delivers that utility efficiently (i.e. at the minimum cost to the firm). Then we turn to period 1 and we solve for the optimal period-1 cash compensation, stock grants (s_H, s_L), and promised future utility levels (U_H, U_L).

3.1 Period 2

First, consider the problem of a firm that retained its manager at the end of period 1. The state variables of this problem are the manager's promised utility U and equity stake s. The manager's consumption will be the sum of cash compensation and dividends, that is $c = w + s(\pi - w) = s\pi + (1-s)w$. The value of the firm at this stage is given by $V_2(s, U)$, where

$$V_2(s, U) = \max_{w_H, w_L} \overline{\rho}\left[\pi_H - w_H\right] + (1-\overline{\rho})\left[\pi_L - w_L\right] \tag{P2}$$

subject to

$$\overline{\rho}u[(1-s)w_H + s\pi_H] + (1-\overline{\rho})u[(1-s)w_L + s\pi_L] - a = U, \tag{1}$$

$$\begin{aligned}&\overline{\rho}u[(1-s)w_H + s\pi_H] + (1-\overline{\rho})u[(1-s)w_L + s\pi_L] - a \geq \\ &\quad \underline{\rho}u[(1-s)w_H + s\pi_H] + (1-\underline{\rho})u[(1-s)w_L + s\pi_L].\end{aligned} \tag{2}$$

Condition (1) is the promise-keeping constraint. It requires that the contract delivers exactly the promised utility U. Condition (2) is the incentive compatibility constraint. Throughout the paper we restrict our attention to scenarios in which it is always optimal for the firm to induce the manager to exert the high level of effort.

Next, consider a firm that fired its manager at the end of period 1 and hence needs to hire a new one at the beginning of period 2. Given that there is an unlimited supply of potential managers, the firm will offer to the new hire exactly his reservation utility ω. Therefore, the firm's outside value equals $V_2(0, \omega)$, i.e. the value of the firm when the manager in charge in period 2 does not hold stock and he is promised an expected utility equal to his outside value. The new manager's compensation contract will consist of a schedule of contingent cash payments only.

The outside value for the manager consists of the utility he expects to receive as of the beginning of period 2, conditional on quitting the firm. This value will depend on the vesting clauses and sale restrictions that apply to the stock grant.

Risk aversion implies that in scenario A a manager that quits will liquidate his position at the beginning of period 2. Therefore, his expected utility is given by

$$\underline{U}(s) = \underline{U}_A(s) = u(c^* + sV_2(0, \omega)). \tag{3}$$

In scenario B, the sale restriction will not allow him to dispose of the stock before the end of the period. This implies that the payoff to quitting is

$$\underline{U}(s) = \underline{U}_B(s) = \overline{\rho}u[c^* + s(\pi_H - w_H^*)] + (1-\overline{\rho})u[c^* + s(\pi_L - w_L^*)], \tag{4}$$

where (w_H^*, w_L^*) are the cash compensations awarded to the newly-hired manager, i.e. the solution to Problem (P2) when $s = 0$ and $U = \omega$. Finally, in scenario C the outgoing manager's expected utility is simply

$$\underline{U}(s) = \underline{U}_C(s) = \omega. \tag{5}$$

3.2 Period 1

At the beginning of period 1, the manager's reservation utility is $u(c^*) + \delta u(c^*) = (1+\delta)\omega$. From our earlier discussion, the firm's task at the beginning of period 1 is to choose the contract $\{(w_i, s_i, U_i),\ i = H, L\}$ that maximizes the value of current shareholders. We recall that here w_i denotes cash compensation in period 1, s_i is the stock grant that the manager will receive at the end of period 1, and U_i defines the manager's expected utility in period 2. All three components of the contract are contingent on the realization of state i in period 1. Therefore, the firm value at the beginning of period 1 is:

$$V_1 = \max_{\{w_i, s_i \in [0,1], U_i\}_{i=H,L}} \overline{\rho}[\pi_H - w_H + \delta(1-s_H)V_2(s_H, U_H)] + (1-\overline{\rho})[\pi_L - w_L + \delta(1-s_L)V_2(s_L, U_L)] \tag{P1}$$

subject to

$$\overline{\rho}\left[u(w_H) + \delta U_H\right] + (1-\overline{\rho})\left[u(w_L) + \delta U_L\right] - a \geq (1+\delta)\omega, \tag{6}$$

$$\begin{aligned} &\overline{\rho}\left[u(w_H) + \delta U_H\right] + (1-\overline{\rho})\left[u(w_L) + \delta U_L\right] - a \geq \\ &\quad \underline{\rho}\left[u(w_H) + \delta U_H\right] + (1-\underline{\rho})\left[u(w_L) + \delta U_L\right], \end{aligned} \tag{7}$$

$$U_i \geq \underline{U}(s_i), \qquad i = H, L, \tag{8}$$

$$V_2(s_i, U_i) \geq V_2(0, \omega), \qquad i = H, L. \tag{9}$$

Conditions (6) and (7) are the individual rationality and incentive compatibility constraints, respectively. Condition (8) imposes that the manager must be offered a continuation utility larger than his period-2 outside value. Condition (9) imposes that the continuation value for the shareholders be greater than their value if they fire the current manager and hire a new one in period 2. Constraints (8) and (9) are the enforceability constraints.

3.3 Enforceability

For a given stock grant s, the only period-2 compensation schedules that the firm can credibly commit to deliver are those that imply expected utility levels U that satisfy conditions (8) and (9). Such couples (s, U) are said to be *enforceable.* The notion of enforceability is now formally introduced.

Definition 2 *A pair* (s, U) *is said to be enforceable if* $U \geq \underline{U}(s)$ *and* $V_2(s, U) \geq V_2(0, \omega)$.

We also define the enforceability correspondence Φ as

$$\Phi(s) \equiv \{U : U \geq \underline{U}(s), V_2(s, U) \geq V_2(0, \omega)\}, \; s \in [0, 1].$$

Alternatively, when $V_2(s, U)$ is strictly decreasing in U, we can write

$$\Phi(s) = \begin{cases} \varnothing & \text{if } \underline{U}(s) > \overline{U}(s), \\ [\underline{U}(s), \overline{U}(s)] & \text{otherwise}, \end{cases}$$

where $\forall\ s$, $\overline{U}(s)$ solves

$$V_2(s, \overline{U}(s)) = V_2(0, \omega). \tag{10}$$

Clearly, when $V_2(s, U)$ is strictly decreasing in U, then for all level of stock holding s, $\overline{U}(s)$ is the highest expected utility the long-term contract can credibly promise to the manager. The correspondence $\Phi(s)$ defines the set of continuation utility values that the firm can credibly promise to its manager, conditional on awarding him an equity stake s. It is immediate that, regardless of the the vesting clause and sale restrictions, $\overline{U}(0) = \underline{U}(0) = \omega$. Further, for all $s \in (0, 1]$, $\underline{U}_A(s) > \underline{U}_B(s) > \underline{U}_C(s)$. In turn, these facts imply directly two properties of the enforceability correspondence, that are stated in the following Lemma.

Lemma 1

1. *For every vesting clause,* $\Phi(0) = \{\omega\}$.
2. $\forall\ s \in [0, 1], \Phi_A(s) \subseteq \Phi_B(s) \subset \Phi_C(s)$.

The second result implies a weak ordering over the firm values in the three vesting scenarios. In Scenario B, firm value will be weakly higher than in Scenario A and weakly lower than in Scenario C. Therefore, a first prescription of our model is that firms are always weakly better off by denying vesting in the event the employee quits his job. In spite of this conclusion, it is still relevant to consider scenarios A and B. In fact these are the only available alternatives if courts are unable to establish which party was responsible for the termination of the contract, so that vesting cannot be made contingent on this event.

In Section 4.2.2 we characterize the enforceability correspondence in the three scenarios and we provide sufficient conditions for non-emptiness.

4 Analysis

In this section we characterize the optimal compensation policy. We begin by analyzing the benchmark scenario in which stock grants are not allowed (i.e. the case of $s = 0$). Then we proceed to consider the more interesting case in which stock grants are used.

4.1 When stock grants are not allowed

Here we show that if the compensation package does not include stock grants, then the long-term contract collapses to a sequence of static contracts. That is, the cash compensation awarded in period 2 does not depend on period-1 profits.[10]

Proposition 1 *If stock grants are not allowed, the optimal dynamic contract collapses to a sequence of static contracts.*

Proof. Consider Problem (P1). Set $s_H = s_L = 0$. Since $\Phi(0) = \{\omega\}$, it must be the case that $U_H = U_L = \omega$. This means that the manager's utility in period 2 does not depend on the first-period outcome. The value of the firm at the beginning of period 1 is then given by

$$V_1^{cash} = \max_{w_H, w_L} \overline{\rho}\left[\pi_H - w_H\right] + (1 - \overline{\rho})\left[\pi_L - w_L\right] + \delta V_2(0, \omega)$$

subject to

$$\begin{aligned}
&\overline{\rho} u(w_H) + (1 - \overline{\rho}) u(w_L) - a = \omega, \\
&\overline{\rho} u(w_H) + (1 - \overline{\rho}) u(w_L) - a \geq \underline{\rho} u(w_H) + (1 - \underline{\rho}) u(w_L).
\end{aligned}$$

We conclude that, in the case of $s \equiv 0$, the feasible sets of the programs (P1) and (P2) are the same and the objective functions differ by an additive constant. Therefore, the maximizers must be the same. The contingent cash compensations are equal across periods. ■

4.2 When stock grants are allowed ($s \geq 0$)

Here we consider the general case in which the firm is allowed to include stock grants in the manager's compensation package. We find it useful to introduce the variable u_i, $i = H, L$. The value u_i denotes the period-2 utility from consumption that the manager receives in state i. We also denote the inverse of the utility function as $v(\cdot)$. That is, we write $v(u) \equiv u^{-1}(u)$.

[10] This result holds in more general setups than ours. For example, see Kocherlakota (1996).

4.2.1 Optimal period-2 compensation

Contingent on continuation of the contract, total firm value $V_2(s,U)$ is given by

$$V_2(s,U) = \max_{u_H,u_L} \overline{\rho}\left[\pi_H - w_H\right] + (1-\overline{\rho})\left[\pi_L - w_L\right],$$

subject to

$$\overline{\rho}u_H + (1-\overline{\rho})u_L - a = U, \tag{11}$$

$$\overline{\rho}u_H + (1-\overline{\rho})u_L - a \geq \underline{\rho}u_H + (1-\underline{\rho})u_L, \tag{12}$$

$$w_H = v(u_H)\frac{1}{1-s} - \frac{s}{1-s}\pi_H, \tag{13}$$

$$w_L = v(u_L)\frac{1}{1-s} - \frac{s}{1-s}\pi_L. \tag{14}$$

Conditions (13) and (14) are derived from the definition of the newly-introduced variable u_i. In fact, we have that $u_i = u(s\pi_i + (1-s)w_i)\ \ \forall\, i$.[11]

Lemma 2 *The incentive compatibility constraint (12) is binding at the optimum.*

Proof. Rewrite condition (12) as $(\overline{\rho}-\underline{\rho})(u_H - u_L) \geq a$. Suppose that at the optimum this constraint holds with strict inequality. Then, it is possible to decrease u_H and increase u_L in such a way that both condition (11) and (12) are satisfied. However, since the inverse of the utility function is strictly convex, the value of the firm is now strictly higher. Obviously, this contradicts the assumption that the starting pair (u_H, u_L) is optimal. ■

In light of Lemma 2, we can use (11) and (12) to solve for the optimal pair (u_H, u_L). We obtain

$$u_H = U + \frac{1-\underline{\rho}}{\overline{\rho}-\underline{\rho}}\,a, \tag{15}$$

$$u_L = U - \frac{\underline{\rho}}{\overline{\rho}-\underline{\rho}}\,a. \tag{16}$$

Substituting (15) and and (16) in the objective function, we obtain that

$$V_2(s,U) = \frac{1}{1-s}\left[\overline{\pi} - f(U;a)\right], \tag{17}$$

[11] Notice that the high effort level $e = a$ is not always implementable. In fact condition (12) implies that $u_H \geq u_L + \frac{a}{\overline{\rho}-\underline{\rho}}$. The latter, together with condition (11), requires that $U \geq u_L + \frac{a\underline{\rho}}{\overline{\rho}-\underline{\rho}}$. Finally, since $w_L \geq 0$, it must hold that $U \geq u(s\pi_L) + \frac{a\underline{\rho}}{\overline{\rho}-\underline{\rho}}$. Therefore a sufficient, albeit not necessary condition for $e = a$ to be implementable for every pair (s,U) such that $U \geq \omega$, is that $\omega \geq u(\pi_L) + \frac{a\underline{\rho}}{\overline{\rho}-\underline{\rho}}$.

where, for every x, the function $f(x;a)$ is defined as

$$f(x;a) \equiv \overline{\rho} v \left(x + \frac{1-\underline{\rho}}{\overline{\rho}-\underline{\rho}} \, a \right) + (1-\overline{\rho}) v \left(x - \frac{\underline{\rho}}{\overline{\rho}-\underline{\rho}} \, a \right).$$

Notice that $f(x;a)$ defines the expected cost to the firm of delivering to the manager the period-2 utility level x, conditional on the recommended effort level being a. It is easy to show that the function f is strictly increasing and strictly convex in x, for any given a.

4.2.2 The enforceability correspondence $\Phi(s)$

By (17), the outside value for the firm at the beginning of period 2 is given by

$$V_2(0,\omega) = \overline{\pi} - f(\omega;a).$$

Therefore, it follows that

$$\overline{U}(s) = f^{-1}\left[f(\omega;a) + s(\overline{\pi} - f(\omega;a)) \right].$$

Given the properties of f, if $\overline{\pi} > f(\omega;a)$, then

$$\overline{U}'(s) > 0 \;\; \forall \, s.$$

This simply says that the utility the firm can commit to deliver to its manager increases monotonically in the size of the stock grant. It is the formal statement of the idea that motivated our work. Accordingly, from now on we will maintain the following assumption:

Assumption 1 $\overline{\pi} > f(\omega;a)$.

Showing that granting stock increases the firm's ability to reward its manager in the future, is not enough. In fact, if $\overline{U}(s) < \underline{U}(s)$ for some s, awarding to the manager an equity stake s will make it optimal for him to breach the contract and quit. We need to provide conditions that insure that the correspondence Φ is nonempty. In Scenario C, it is clear that this is always the case for all s, since $\underline{U}_C(s) = \omega \;\; \forall s \in [0,1]$. In scenarios A and B, non-emptiness of Φ is not a general property.

Since the value function $V_2(s,U)$ is strictly decreasing in U, $\Phi(s)$ is not empty if and only if

$$V_2(s,\underline{U}(s)) \geq V_2(0,\omega). \tag{18}$$

In turn, condition (17) implies that the latter holds if and only if

$$f(\underline{U}(s); a) - s[\overline{\pi} - f(\omega; a)] \leq f(\omega; a). \tag{19}$$

Proposition 2 states a condition under which, in both scenarios A and B, the set $\Phi(s)$ is non-empty in an interval that includes $\{0\}$.

Proposition 2 *If*

$$\overline{\rho} v' \left(\omega + \frac{1-\rho}{\overline{\rho} - \underline{\rho}}\, a \right) + (1 - \overline{\rho}) v' \left(\omega - \frac{\rho}{\overline{\rho} - \underline{\rho}}\, a \right) < v'(\omega), \tag{20}$$

then there exist values s_A, s_B, $0 < s_A < s_B \leq 1$, *such that* $\Phi_A(s)$ *is non-empty over* $[0, s_A]$ *and* $\Phi_B(s)$ *is non-empty over* $[0, s_B]$.

Proof. Consider case A first. For $s = 0$, condition (19) holds with equality. Therefore, we just need to show that

$$f(\underline{U}_A(s); a) - s[\overline{\pi} - f(\omega; a)] \tag{21}$$

is decreasing in s at $s = 0$. The first derivative of (21) with respect to s, evaluated at $s = 0$, is given by

$$V_2(0, \omega) \left\{ u'(c^*) \left[\overline{\rho} v' \left(\omega + \frac{1-\rho}{\overline{\rho} - \underline{\rho}}\, a \right) + (1 - \overline{\rho}) v' \left(\omega - \frac{\rho}{\overline{\rho} - \underline{\rho}}\, a \right) \right] - 1 \right\}. \tag{22}$$

Given our assumptions on the utility function, the derivative is a continuous function. Therefore, if (22) is negative, there exists $s_A > 0$ such that $\Phi_A(s)$ is non-empty over $[0, s_A]$. Finally, this is equivalent to requiring that

$$\overline{\rho} v' \left(\omega + \frac{1-\rho}{\overline{\rho} - \underline{\rho}}\, a \right) + (1 - \overline{\rho}) v' \left(\omega - \frac{\rho}{\overline{\rho} - \underline{\rho}}\, a \right) < v'(\omega).$$

Since f is a strictly increasing function and $\underline{U}_B(s) < \underline{U}_A(s)\ \forall\ s$, it follows that for all $s > 0$ such that $\Phi_A(s)$ is non-empty, $\Phi_A(s) \subset \Phi_B(s)$. Further, there is a non-empty interval $(s_A, s_B]$ such that $\Phi_A(s) = \varnothing$ and $\Phi_B(s) \neq \varnothing\ \forall\ s \in (s_A, s_B]$. ■

In Figure 2, we plot the difference $\overline{U}(s) - \underline{U}(s)$ in a parametric case in which $u(c) = c^{1-\sigma}$, $\sigma < .5$, so that condition (20) is satisfied. Notice that $s_A < 1$ and $s_B = 1$.

When condition (20) does not hold, it can be the case that the enforceability correspondence is empty on an interval immediately to the right of $s = 0$, but it is non-empty for larger values of s.[12]

[12] By the same argument used in the proof of Proposition 2, it is easy to show that when $v'(\cdot)$ is strictly convex, the enforceability correspondence is empty for s close enough to 0, in both scenarios.

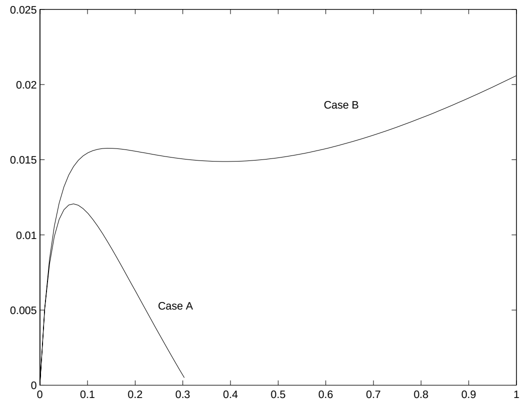

Figure 2: $\overline{U}(s) - \underline{U}(s)$

4.2.3 The firm's problem in period 1

Let now V_1^{stock} denote the value of the firm at the beginning of period 1. Such value is given by

$$V_1^{stock} = \max_{\{u_i, s_i, U_i\}_{i=H,L}} \overline{\rho}[\pi_H - v(u_H) + \delta(1 - s_H)V_2(s_H, U_H)] + \\ (1 - \overline{\rho})[\pi_L - v(w_L) + \delta(1 - s_L)V_2(s_L, U_L)]$$

subject to

$$\overline{\rho}[u_H + \delta U_H] + (1 - \overline{\rho})[u_L + \delta U_L] - a = (1 + \delta)\omega, \tag{23}$$

$$\overline{\rho}[u_H + \delta U_H] + (1 - \overline{\rho})[u_L + \delta U_L] - a \geq \underline{\rho}[u_H + \delta U_H] + (1 - \underline{\rho})[u_L + \delta U_L], \tag{24}$$

$$U_i \in \Phi(s_i), \ i = H, L. \tag{25}$$

Using (17), it is easy to show that

$$V_1^{stock} = - \min_{\{u_i, s_i, U_i\}_{i=H,L}} \overline{\rho}\,[v(u_H) + \delta f(U_H; a)] + (1 - \overline{\rho})\,[v(u_L) + \delta f(U_L; a)] - (1 + \delta)\overline{\pi}$$
$$\text{subject to (23), (24), (25).}$$

Proposition 3 is our main result. It shows that the ability to award stock grants allows the firm to partially overcome its lack of commitment. Including stock grants in the compensation package allows to make period-2 compensation contingent on first-period outcomes, thereby increasing shareholder value. Proposition 3 applies to

Lemma 3 in Appendix B shows that in scenario A strict concavity of $v'(\cdot)$ is actually necessary for non emptiness, for all s. This implies, for example, that if $u(c) = log(c)$, then $\Phi_A(s) = \varnothing$ for all $s \in (0, 1]$. In Lemma 4 we give a necessary condition under which $\Phi_B(s) \neq \emptyset$ for any fixed $s > 0$ for a class of utility functions including $u(c) = log(c)$.

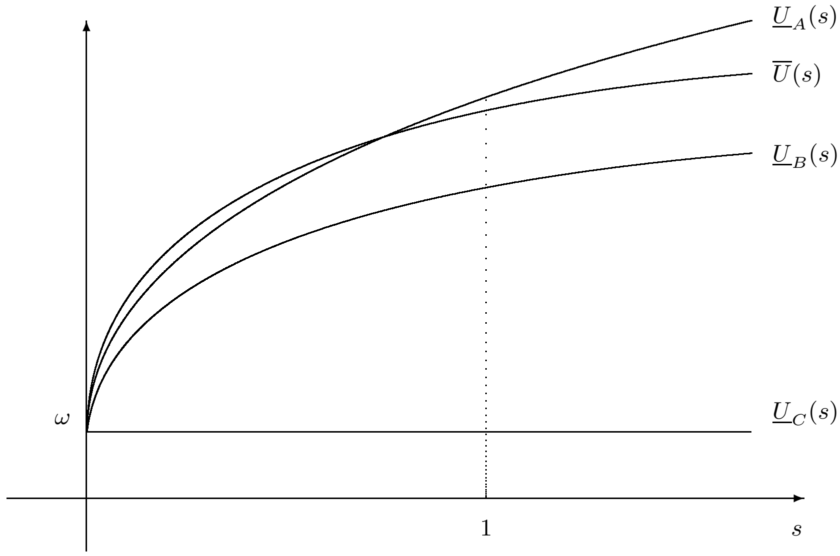

Figure 3: Enforceability Correspondence.

all cases in which $\Phi(s)$ is nonempty on an interval $[0, s^*]$, for some $s^* \in (0,1]$. In scenario C, this is always true. By Proposition 2, we know that when condition (20) holds, it is true regardless of the vesting clause. Figure 3 presents a typical case in which the assumption of Proposition 3 is satisfied.

The analysis of the optimal contract in the case in which $\Phi(s) = \varnothing$ in an interval immediately to the right of $s = 0$ is included in Appendix A. It turns out that the results stated in Proposition 3 hold true also in that case, provided that a further condition is imposed.

Proposition 3 *If $\Phi(s)$ is non-empty over $[0, s^*]$ for $s^* \in (0,1]$, then the following conditions are necessary for optimality:*

1. $U_L = \omega$,
2. $U_H > \omega$,
3. $s_H > 0$.

Proof. Since the function $f(x; a)$ is strictly convex in x, one can use the argument used in the proof of Lemma 2 to show that, in solution, condition (24) holds with equality. Therefore, using (23) and (24), we get that

$$u_H + \delta U_H = \omega(1+\delta) + \frac{1-\underline{\rho}}{\overline{\rho}-\underline{\rho}}\, a$$
$$u_L + \delta U_L = \omega(1+\delta) - \frac{\underline{\rho}}{\overline{\rho}-\underline{\rho}}\, a$$

Then, necessary and sufficient conditions for a maximum are:

$$v'[\omega(1+\delta) + \frac{1-\underline{\rho}}{\overline{\rho}-\underline{\rho}}\,a - \delta U_H] - \frac{\partial f(U_H;a)}{\partial U_H} \leq 0 \tag{26}$$

$$v'[\omega(1+\delta) - \frac{\underline{\rho}}{\overline{\rho}-\underline{\rho}}\,a - \delta U_L] - \frac{\partial f(U_L;a)}{\partial U_L} \leq 0 \tag{27}$$

Notice that the left hand side of (27) is a monotone decreasing function of U_L. It is straightforward to show that for $U_L = \omega$ such function assumes a strictly negative value. This implies that it is optimal to choose $U_L = \omega$. The left-hand side of (26) is also strictly decreasing in U_H and it is immediate to show that for $U_H = \omega$, it assumes a strictly positive value. Therefore in solution $U_H > \omega$. Then, enforceability also implies that $s_H > 0$. In fact, $\overline{U}(0) = \omega$. ■

A corollary of Proposition 3 is that the value of the firm is now higher than in Section 4.2. This is the case, because the possibility of awarding stock grants enables the firm to use both current and deferred compensation as incentive devices in Period 1. For given outside value ω, a positive spread between U_H and U_L implies a lower spread between u_H and u_L. In turn, by strict concavity of the utility function, this implies a lower expected compensation, and thus higher firm value.

Corollary 1 $V_1^{stock} > V_1^{cash}$.

Proof. The value V_1^{cash} is the value of Problem (P1) under the restriction that $s_H = s_L = 0$. This proves that $V_1^{cash} \leq V_1^{stock}$. The fact that the inequality is strict follows from the observation that Problem (P1) defines the maximization of a strictly concave function over a strictly convex set, and therefore admits only one maximizer. ■

By Lemma 1, we know that firm value in Scenario B will be weakly higher than in scenario A and weakly lower than in Scenario C. Notice, however, that if in scenario A the upper bound s^* is so large that the constraint $U_H \leq \overline{U}(s^*)$ does not bind, then the optimal compensation contract (and therefore firm value) is the same across the three vesting scenarios.

5 Stock Option Grants

In the previous section we have established that in an environment characterized by imperfect contractual enforcement, including stock grants in the compensation packages has the potential to increase shareholder value.

This results depends crucially on the assumption that the firm can commit to actually grant stock at the end of period 1. In this section we investigate how the contract changes once we dispense with this hypothesis. We consider the case in which the firm replaces the contingent stock grant with options awarded at the beginning of time (i.e. when the contract is signed). Under the new circumstance, the payoff accruing to the manager from the option grant still depends on the firm's performance in the first period, but it is not subject to the owners' discretion.

We assume that the compensation contract consists of cash compensation and a stock option grant (z, P), to be awarded at the beginning of time and exercisable at the end of period 1. With z we denote the largest fraction of the firm that the manager can acquire, should he decide to exercise. In other words, z is the size of the option grant, measured as a percentage of total equity. We denote the exercise price (or, rather, the exercise total value) of the option, as P. Notice that once the manager has decided how many options to exercise, the firm's problem is the same as in the previous sections. This means that we can focus on period-1 problem. The value of the firm is given by

$$V_1^{opt} = \max_{z,P,\{w_i,s_i\in[0,z],U_i(s)\}_{i=H,L}} \overline{\rho}[\pi_H - w_H + \delta(1-s_H)V_2(s_H,U_H)] + (1-\overline{\rho})[\pi_L - w_L + \delta(1-s_L)V_2(s_L,U_L)] \tag{P3}$$

subject to

$$\overline{\rho}\left[u(w_H - s_H P) + \delta U_H(s_H)\right] + (1-\overline{\rho})\left[u(w_L - s_L P) + \delta U_L(s_L)\right] - a = (1+\delta)\omega, \tag{28}$$

$$\overline{\rho}\left[u(w_H - s_H P) + \delta U_H(s_H)\right] + (1-\overline{\rho})\left[u(w_L - s_L P) + \delta U_L(s_L)\right] - a \geq \underline{\rho}\left[u(w_H - s_H P) + \delta U_H(s_H)\right] + (1-\underline{\rho})\left[u(w_L - s_L P) + \delta U_L(s_L)\right], \tag{29}$$

$$s_H = \arg\max_{s\in[0,z]} u(w_H - sP) + \delta U_H(s), \tag{30}$$

$$s_L = \arg\max_{s\in[0,z]} u(w_L - sP) + \delta U_L(s), \tag{31}$$

$$U_i(s) \geq \underline{U}(s), \quad i = H, L, \ s \in [0, z], \tag{32}$$

$$V_2(s, U_i(s)) \geq V_2(0, \omega), \qquad i = H, L, \ s \in [0, z]. \tag{33}$$

The firm chooses the option grant (z, P) and the amount of options $s_i \in [0, z]$ that it wishes the manager to exercise, should state i occur. Constraints (30) and (31) impose that it is optimal for the manager to exercise s_i. The variable $U_i(s)$ is the expected continuation utility awarded to the manager, should state i occur, and should the manager exercise exactly s options. Conditions (32) and (33) impose that all promised utilities are enforceable, both *on* and *off* the equilibrium path.

A relevant feature of this contract is that period-2 wages are conditional on the exercise decision of the manager. That is, conditional on a given realization (high or low), two managers that exercise different quantities of their options may end up receiving different cash wages in period 2. It is the nature of the optimal contract that the cash payments can be adjusted according to realizations of the states. This may seem a little unusual but we would argue that it is not. It is not uncommon for managers whose options finish out of the money to have compensation adjusted in subsequent periods. This is the spirit of the discussion in Acharya et al. (2000).

5.1 Analysis

For simplicity, we limit our analysis to Scenario A. Notice first that the continuation utilities off the equilibrium path do not enter the objective function or any of the constraints, except for conditions (30) and (31). Therefore, without loss of generality, off the equilibrium path we can set $U_i(s) = \underline{U}(s),\ \forall s$. That is, if the manager deviates from the suggested exercise policy, he will get his outside value. For any i, let also:

$$s(w_i, P) \equiv \arg\max_{s\in[0,z]} u(w_i - sP) + \delta \underline{U}(s).$$

The quantity $s(w_i, P)$ is the fraction of options that the manager exercises if faced with a first-period wage w_i, exercise price P, and continuation utility $\underline{U}(s)$. In other words, $s(w_i, P)$ is the manager's optimal deviation. As in the previous sections, let $u_i \equiv u(w_i - s_i P)$. Since the incentive compatibility constraint (29) binds, we can use (28) and (29) to obtain

$$u_H + \delta U_H(s_H) = (1+\delta)\omega + \frac{1-\underline{\rho}}{\overline{\rho}-\underline{\rho}}\, a,$$
$$u_L + \delta U_L(s_L) = (1+\delta)\omega - \frac{\underline{\rho}}{\overline{\rho}-\underline{\rho}}\, a.$$

Finally, again without loss of generality, let $z = max(s_H, s_L)$. Then, the problem of the firm can be rewritten as follows:

$$V_1^{opt} = -\min_{P,\{s_i\in[0,z],U_i\}_{i=H,L}} \overline{\rho}\left[v(u_H) + \delta f(U_H; a)\right] + (1-\overline{\rho})\left[v(u_L) + \delta f(U_L; a)\right] - (1+\delta)\overline{\pi},$$

subject to

$$U_i \in \Phi(s_i),\quad i = H, L,$$

$$u(w_H - s(w_H, P)P) + \delta \underline{U}(s(w_H, P)) \leq (1+\delta)\omega + \frac{1-\underline{\rho}}{\overline{\rho}-\underline{\rho}}\, a, \tag{34}$$

$$u(w_L - s(w_L, P)P) + \delta \underline{U}(s(w_L, P)) \leq (1+\delta)\omega - \frac{\underline{\rho}}{\overline{\rho}-\underline{\rho}}\, a, \tag{35}$$

$$u_H = (1+\delta)\omega + \frac{1-\underline{\rho}}{\overline{\rho}-\underline{\rho}}\, a - \delta U_H,$$
$$u_L = (1+\delta)\omega - \frac{\underline{\rho}}{\overline{\rho}-\underline{\rho}}\, a - \delta U_L,$$

$$w_H = s_H P + v\left[(1+\delta)\omega + \frac{1-\underline{\rho}}{\overline{\rho}-\underline{\rho}}\, a - \delta U_H)\right],$$
$$w_L = s_L P + v\left[(1+\delta)\omega - \frac{\underline{\rho}}{\overline{\rho}-\underline{\rho}}\, a - \delta U_L)\right].$$

Notice that conditions (34) and (35) are the reformulation of conditions (30) and (31). Proposition 4 states that the contract with stock options implies a firm value that is lower than the contract with stock grants. The intuition is simple. In order to replicate the allocation achieved with stock grants, the compensation contract with options must be such that in the high state the manager exercises a quantity of options equal to the stock grant. In the low state, instead, he must find it convenient not to exercise any option. In general such a contract does not exist. Proposition 4 states also that the contract with options performs strictly better than the contract with cash compensation only. The reason is that, contrary to the contract with cash only, the contract with options allows continuation utilities U_H and U_L to differ.

Proposition 4 *The values V_1^{cash}, V_1^{stock}, V_1^{opt} satisfy $V_1^{cash} < V_1^{opt} \leq V_1^{stock}$.*

Proof. The fact that $V_1^{opt} \leq V_1^{stock}$ follows directly from the observation that V_1^{opt} maximizes the same function as V_1^{stock}, but on a smaller feasibility set. To prove that $V_1^{cash} < V_1^{opt}$, we will show that there exists a feasible, and possibly sub-optimal contract with options, that delivers a firm value strictly larger than V_1^{cash}. Let $s_L = 0$ and $U_L = \omega$, so that $w_L = v\left(\omega + \frac{\underline{\rho}}{\overline{\rho}-\underline{\rho}}\, a\right)$. Such choices imply that, conditional on the low state occurring, the manager will receive the same utility awarded by the optimal contract with cash. Further, let $P = \delta \frac{V_2(0,\omega)u'(c^*)}{u'(w_L)}$. At such exercise price, it is optimal for the manager to exercise zero options in the low state. Finally, whatever s_H, set $U_H = \underline{U}(s_H)$. Now we just need to show that there exists a couple (w_H, s_H), with $s_H > 0$, such that the two following conditions hold:

$$u(w_H - s_H P) + \delta \underline{U}(s_H) = (1+\delta)\omega + \frac{1-\rho}{\overline{\rho} - \underline{\rho}}\, a, \tag{36}$$

$$-Pu'(w_H - s_H P) + \delta V_2(0,\omega) u'(c^* + s_H V_2(0,\omega)) \geq 0. \tag{37}$$

Condition (37) requires that it is optimal for the manager to exercise s_H options in the high state. Recall that $u(w_L) + \delta \underline{U}(0) = (1+\delta)\omega - \frac{\rho}{\overline{\rho}-\underline{\rho}}\, a$. Therefore, there exists a value w_H, with $w_H > w_L$, such that $u(w_H) + \delta \underline{U}(0) = (1+\delta)\omega + \frac{1-\rho}{\overline{\rho}-\underline{\rho}}\, a$. Given that $u' > 0, u'' < 0$, it follows that $-Pu'(w_H) + \delta V_2(0,\omega)u'(c^*) > 0$. Then, by continuity of u', there exist strictly positive values s_H, and wages $w_H(s_H)$ implied by (36), such that the pairs $(s_H, w_H(s_H))$ satisfy (37). ■

The main lesson of Proposition 4 is that, even if enforcement problems are so severe that contingent grants of stock are not available, awarding call options has still the potential to increase shareholder value. Proposition 4 also says that, in general, shareholder value is higher when stock is granted. Obviously this does not imply that in reality firms should always reward managers with stock bonus plans instead of option grants. There are other factors, not considered in our model, that might make options more appealing than stock. In particular, we refer to the differential tax and accounting treatment of stock and option grants. Our analysis just points out that when considering the role of securities grants as a partial solution to the commitment problem generated by imperfect enforcement of compensation contracts, option grants do not have any advantage over stock grants.

6 Conclusion

In this paper we have presented a simple model of the relationship between a firm and its manager. We have shown that if the enforcement of compensation contracts is imperfect, cash compensation and stock grants are no longer perfectly substitutable means to compensate managers. By awarding stock, a company is able to overcome (at least partially) its lack of commitment and can credibly promise to deliver continuation utility levels that are higher than the manager's reservation utility. As a consequence, deferred compensation can be made contingent on current performance. By using both current and deferred compensation for incentive purposes, a firm can provide its manager with a given utility level at a lower cost, therefore increasing shareholder value. Our analysis also shows that if the commitment problems generated by imperfect enforcement are so severe so as to make contingent stock grants unavailable, firms can still improve over only-cash compensation, by awarding options. In our environment, stock grants outperform call options as a way to discipline managers. However, our setup omits many of the relevant features which may be important for preferring options to stock grants.

In the introduction we have argued that the literature on the design of optimal compensation contracts is still in its infancy. It is our opinion that further work in the area is warranted, as it could be useful to both companies' compensation committees and regulators. In particular, we think it would be of interest to study environments where the manager not only chooses effort, but also the riskiness of its projects and/or the information flow reaching the investors.[13]

[13]Carpenter (2000) studies the investment problem of a risk-averse manager compensated with options. However, options are not optimal in her environment.

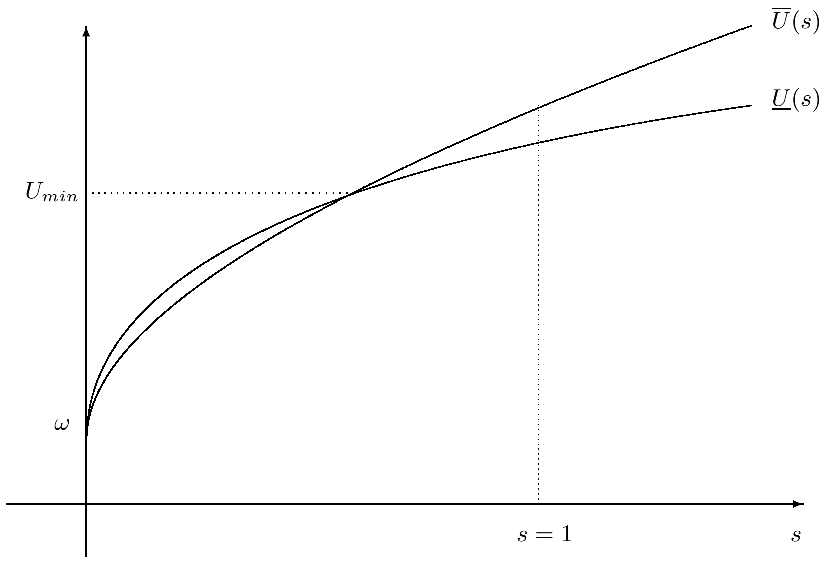

Figure 4: Enforceability Correspondence

A Optimal Contract with Stock: Complete Analysis

In this appendix, we generalize our result in Proposition 3 to the cases where $\Phi(s)$ is not necessarily non-empty over $[0, s^*]$ for some $s^* \in (0, 1]$. One such case is depicted in Figure 4.

Let $\tilde{\Phi} \equiv \{U : \exists\ s \in [0,1]$ *such that* $U \in \Phi(s).\}$ That is, $\tilde{\Phi}$ is the set of all values of U that are enforceable (together with some $s \in [0,1]$). Finally, let $U_{min} \equiv \min\{U : U \in \tilde{\Phi},\ U \neq \omega\}$. In other words, except ω, U_{min} is the lowest enforceable U.

Proposition 5 *(i) The optimal contract has* $U_L = \omega$. *(ii) Suppose* U_{min} *is sufficiently close to* ω *(in the sense to be made precise in the proof). Then the optimal contract must also have* $s_H > 0$ *and* $U_H > \omega$.

Proof. The proof of (i) is the same as in Proposition 3. To prove (ii), notice that

$$U_H = \arg \min_{U_H \in \tilde{\Phi} \bigcup \{\omega\}} F(U_H) \equiv u^{-1}\left(\omega(1+\delta) + \frac{1-\rho}{\overline{\rho} - \underline{\rho}}\, a - \delta U_H\right) + \delta f(U_H; a).$$

Since the function F is strictly convex and strictly decreasing at ω, it follows that $U_H > \omega$ if and only if $F(U_{min}) \leq F(\omega)$. Finally, the enforceability condition $U_H \in \Phi(s_H)$ implies $s_H > 0$. ■

We now reconsider the numerical example introduced in Section 4.2.2 where $u(c) = \log(c)$, $\delta = 0.75$, $\pi_L = 0.01$, $\pi_H = 1.0$, $\underline{\rho} = 0.0$, $\overline{\rho} = 0.5$, $a = 1.0$, $\omega = \log(0.0001) = -9.21$. Figure 5 plots the function $F(U_H)$. Clearly, in this example, $F(U_{min}) \leq F(\omega)$. Therefore we have $s_H > 0$ at the optimum.

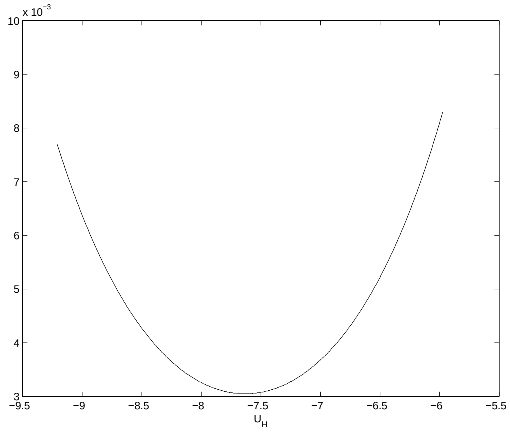

Figure 5: The Function $F(U_H)$

B Lemmas

Lemma 3 *If* $\forall\ e \in [0, a]$

$$\frac{v'\left[\underline{U}(s) + \frac{1-\rho}{\overline{\rho}-\underline{\rho}}\ e\right] - v'\left[\omega + \frac{1-\rho}{\overline{\rho}-\underline{\rho}}\ e\right]}{v'\left[\underline{U}(s) - \frac{\rho}{\overline{\rho}-\underline{\rho}}\ e\right] - v'\left[\omega - \frac{\rho}{\overline{\rho}-\underline{\rho}}\ e\right]} > \frac{\underline{\rho}(1-\overline{\rho})}{\overline{\rho}(1-\underline{\rho})},$$

then $\Phi_A(s) = \varnothing$.

Proof. By (17), $\underline{U}(s)$ satisfies

$$v\left(\underline{U}(s)\right) = c^* + s\left[\overline{\pi} - f(\omega; a)\right],$$

which is equivalent to

$$f\left(\underline{U}(s); 0\right) - f\left(\omega; 0\right) = s\left[\overline{\pi} - f(\omega; a)\right].$$

On the other hand, $\Phi_A(s) = \varnothing$ if and only if

$$f\left(\underline{U}(s); a\right) - f\left(\omega; a\right) > s\left[\overline{\pi} - f(\omega; a)\right].$$

Sufficient condition for this is that

$$\frac{\partial}{\partial e}\left[f(\underline{U}(s); e) - f(\omega; e)\right] > 0\ \ \forall\ e. \tag{38}$$

It turns out that

$$\frac{\partial f(x; e)}{\partial e} = \overline{\rho}\frac{1-\rho}{\overline{\rho}-\underline{\rho}}\ v'\left(x + \frac{1-\rho}{\overline{\rho}-\underline{\rho}}\ e\right) - (1-\overline{\rho})\frac{\rho}{\overline{\rho}-\underline{\rho}}\ v'\left(x - \frac{\rho}{\overline{\rho}-\underline{\rho}}\ e\right).$$

Then, since $\overline{\rho} - \underline{\rho} > 0$ and $v'' > 0$, (38) holds if and only if

$$\frac{v'\left[\underline{U}(s) + \frac{1-\rho}{\overline{\rho}-\underline{\rho}}\, e\right] - v'\left[\omega + \frac{1-\rho}{\overline{\rho}-\underline{\rho}}\, e\right]}{v'\left[\underline{U}(s) - \frac{\rho}{\overline{\rho}-\underline{\rho}}\, e\right] - v'\left[\omega - \frac{\rho}{\overline{\rho}-\underline{\rho}}\, e\right]} > \frac{\underline{\rho}(1-\overline{\rho})}{\overline{\rho}(1-\underline{\rho})}.$$

■

Lemma 4 *Assume* $\lim_{c\to 0} u(c) = -\infty$. *Then, for any* $s \in (0,1]$ *there exist* $c^* > 0$ *and* $\pi_L > 0$ *such that* $\Phi_B(s) \neq \emptyset$.

Proof. Consider any $s \in (0,1]$. Using (14) and (16), it is easy to obtain

$$\pi_L - w_L^* = \frac{1}{1-s}\left[\pi_L - v\left(\omega - \frac{\rho}{\overline{\rho}-\underline{\rho}}\, a\right)\right].$$

Now recall that by (4),

$$\underline{U}_B(s) = \overline{\rho} u[c^* + s(\pi_H - w_H^*)] + (1-\overline{\rho}) u[c^* + s(\pi_L - w_L^*)].$$

Since $\lim_{u\to -\infty} v(u) = 0$, for every $\overline{u} > -\infty$ there exist $c^* > 0$ and $\pi_L > 0$ such that $\underline{U}_B(s) < \overline{u}$.

On the other hand, it is also the case that $\lim_{\omega\to -\infty} f(\omega; a) = 0$. In turn, this implies that, for any couple (ω, π_L),

$$\overline{U}(s) = f^{-1}[f(\omega; a) + s(\bar{\pi} - f(\omega; a))] > f^{-1}(s\overline{\rho}\pi_H).$$

Therefore, it is enough to pick values $c^* > 0$ and $\pi_L > 0$ such that $\underline{U}_B(s) < f^{-1}(s\overline{\rho}\pi_H)$. ■

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